Exercise 13

Find the differential of each function.

(a) $y = \tan\sqrt{t}$ (b) $y = \dfrac{1-v^2}{1+v^2}$

Solution

Part (a)

Compute the derivative of y.

$$\begin{aligned}\frac{dy}{dt} &= \frac{d}{dt}\left(\tan\sqrt{t}\right)\\ &= \left(\sec^2\sqrt{t}\right)\cdot\frac{d}{dt}(\sqrt{t})\\ &= \left(\frac{1}{\cos^2\sqrt{t}}\right)\cdot\frac{1}{2}t^{-1/2}\\ &= \frac{1}{2\sqrt{t}\cos^2\sqrt{t}}\end{aligned}$$

Therefore, the differential of $y = \tan\sqrt{t}$ is

$$dy = \frac{1}{2\sqrt{t}\cos^2\sqrt{t}}\,dt.$$

Part (b)

Compute the derivative of y.

$$\begin{aligned}\frac{dy}{dv} &= \frac{d}{dv}\left(\frac{1-v^2}{1+v^2}\right)\\ &= \frac{\left[\frac{d}{dv}(1-v^2)\right](1+v^2) - \left[\frac{d}{dv}(1+v^2)\right](1-v^2)}{(1+v^2)^2}\\ &= \frac{(-2v)(1+v^2)-(2v)(1-v^2)}{(1+v^2)^2}\\ &= \frac{-4v}{(1+v^2)^2}\end{aligned}$$

Therefore, the differential of $y = (1-v^2)/(1+v^2)$ is

$$dy = \frac{-4v}{(1+v^2)^2}\,dv.$$